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#8 Let $z=f(x,y)$ By using the general equation for the surface area of a parametric curve, show that the surface area of f over D is

$$\iint_D \sqrt{1+f_x(x,y)^2+f_y(x,y)^2}\, dA$$

parametrize $x=x$ $y=y$ $z=f(x,y)$

$r(x,y)=\langle x, y, f(x,y)\rangle$

$r_x = \langle 1, 0, f_x(x,y)\rangle$

$r_y = \langle 0, 1, f_y(x,y)\rangle$

$$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & f_x(x,y) \\ 0 & 1 & f_y(x,y) \end{vmatrix} = \langle -f_x(x,y), -f_y(x,y), 1\rangle$$

$$|r_x \times r_y| = \sqrt{1+(-f_x(x,y))^2+(-f_y(x,y))^2}$$

$$A = \iint_D |r_x \times r_y|\, dA$$

replace $(r_x \times r_y)$ with solved value

$$\iint_D \sqrt{1+f_x(x,y)^2+f_y(x,y)^2}\, dA$$

and same as top